STUDY GUIDE:

Module 11: Selected Topics in Non-Constant Rates

While constant rates lend themselves to a simple application of ratios, the fact is that many of the most important rates are not constant. What is true, however, is that any study of non-constant rates can be viewed as an outgrowth of the constant rates. In this module we look at three special categories of non-constant rates. Namely:

(1) Mixtures Involving Different Constant Rates

(2) Perimeters, Areas, and Volumes

(3) Exponential Growth

Because areas, volumes, and perimeters involve geometric concepts and because geometry is not actually a part of this course, a special optional videotaped lecture (Lecture 11B) on these topics is included as part of this Module.

Step 1:

View Videotape Lecture #11.

At some point during the study of this module you should also view Lecture #11B if you want additional enrichment in the topics of perimeter, area, and volume.

Step 2:

Read Module 11 of the text.

Step 3:

When you feel you understand the material presented in Module 11, complete the following "Check-The-Main-Ideas" self-quiz on the next page.

Check The Main Ideas:

In this module we deal with ___-constant rates. One category of non-constant rates is when we deal with mixtures containing two or more _____ rates. For example, if you buy shirts at the constant rate of $30 per shirt, 40 shirts would cost $_____. At $20 per shirt, 60 sheets would cost $_____. Hence the total cost of these 100 shirts would be $_____. Therefore the average cost per shirt is $_____. In other words, the average cost per shirt is not the same as the average of $20 and $30, which is $_____.

non

constant

1,200

1,200

2,400

24

25

Another example of mixed constant rates occurs when we average our test grades. For example, if you took 4 tests and scored 75 points on each, you'd have a total of ____ points. In fact, any combination of points adding up to 300 in 4 tests would give you an average score of _____ points. So if on the first three tests you scored 80 points, 91 points, and 83 points, all you'd need on the fourth test to average 75 points is _____ points. Had you scored 70 points in each of the first three tests, then to average 75 points per test you'd have to score _____ points on the fourth test; even though the average of 70 and 90 is _____.

300

75

46

90

80

A second category of non-constant rates occurs in geometry when we study amounts of space. The distance around the boundary of a closed region is called the _____ of the region. To find the perimeter of a rectangle we can _____ the length and the width, and then multiply by _____. For example, since 40 + 70 = 110 and 110 X 2 = 220, the perimeter of 40 foot by 70 foot rectangle is _____. If the rectangle happens to be a square we can find its perimeter by multiplying the length of any side by _____.

perimeter

add

2

220 feet (Don't omit the label)

4

To find the area of a ractangle we take the _____ of the length and the width. For example, since 40 X 70 = 2,800 the area of a 40 foot by 70 foot ractangle is _____. In particular, to find the area of a square we need only _____ one of the sides. For example since $4^2 = 16$, the area of a 4 foot square is _____. If we double the length of the side of a 4 foot square we get an 8 foot square whose area is _____ square feet. That is, when we multiplied the length of each side of the square by 2 we multiplied the area of the square by _____.

product

2,800 square feet

square

16 square feet

64

4

A more complicated problem occurs when we're given the area of the square and we want to find the length of each side. For example, the area of a 10 inch square is _____ square inches,

10

and the area of an 11 inch square is _____ square inches. Hence if the area of a square is 115 square inches, the length of each side is ____ than 10 inches but _____ than 11 inches. Th exact number we want is called the square _____ of 115 and is written as _____. In other words, by its definition $\sqrt{115} \times \sqrt{115} =$ _____. Squares and square roots are related in the same way as are addition and _____. For example the fact that $8^2 = 64$ means that we can either say that the _____ of 8 is 64 or that the _____ of 64 is 8. In terms of the area of a square we square when we're given the _____ and we want to find its _____. We use square roots when we're given the _____ and we want to find the length of each side.

121

more (greater)

less

root

$\sqrt{115}$

115

subtraction

square

square root

length (of each side)

area

area

A third category of non-constant rate occurs when we deal with _____ interest. For example at a 6% annual rate of interest, at the end of 1 year, a $100 deposit would be worth $_____. That is, after 1 year you have _____% of your investment. If the interest is compounded annually, the next year the 6% would not be based on the $100 deposit but rather on $_____. That is, the next year, the value of your deposit will be 106% of $_____. Since $1.06^2 = 1.1236$, after 2 years the value of your $100 deposit will be $_____. That is, over the two year period the value of your $100 has increased by $_____.

compound

106

106

106

106

112.36

12.36

Since you had $106 after 1 year and $112.36 after two years, your deposit earned $_____ during the second year. Hence, based on the original $100 deposit, the second year's earnings were ____% of $100. That is, with compound interest, the amount your deposit earns increases each year. So this is an example of a _____ rate of increase. This is why banks talk about APR or "actual percent rate". For example, your $100 investment earned $12.36 which is a ____% increase; but since this was earned over a two year period, the average percent per year is 12.36% ÷ ___ or ____%. This is called the APR because it represents what the equivalent non-compounded annual interest rate would have had to be in order to give you the same earnings.

6.36

6.36

non-constant

12.36

2; 6.18

The APR increases dramatically as time goes on. For example $1.06^{10} \doteq 1.79$. Hence if you deposited $100 at an interest of 6% compounded annually, after 10 years its value would be $_____. So during this 10 year period your $100 deposit has earned $_____ or ____% of $100. Since this took 10 years the average increase per year was _____% So the APR in this case is _____%. That is, if the interest wasn't compounded you'd need a ____% annual rate to earn $79 in ten years.

179

79; 79

7.9

7.9

7.9

****

Step 4:

Do the Mastery Review on the next page.

Mastery Review:

Answers:

1. A merchant buys 20 coats at $50 per coat and 80 coats at $70. What was the total cost of the 100 coats?

1. _____

2. A merchant buys 20 coats at $50 per coat; 30 coats at $60 per coat; and 50 coats at $70 per coat.

   (a) What was the total cost of the coats?

   (b) What was the average cost per coat?

2. (a) _____

   (b) _____

3. In each of 6 tests you score 83 points. How many points did you get as a total for these 6 tests?

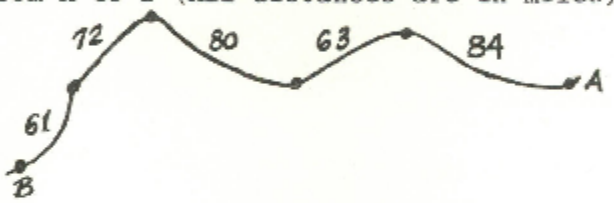
3. _____

4. You take 5 tests and get scores of 78, 80, 81, 82, and 84. How many points must you get on the next test in order to have an average of 83 points per test for the 6 tests?

4. _____

5. Based on the diagram below, how far is it from A to B (All distances are in miles).

5. _____



6. With A and B as in Problem 5, what must your average speed be if you want to make the trip from A to B in 8 hours?

6. _____

7. Referring to Problems 5 and 6, suppose you know that for this kind of trip you car will give you 18 miles per gallon of gas. How many gallons of gas must you use for the trip?

7. _____

8. Under the same conditions as in Problems 5, 6, and 7, suppose you pay $1.24 per gallon of gas. How much will you pay for the gas used during this trip?

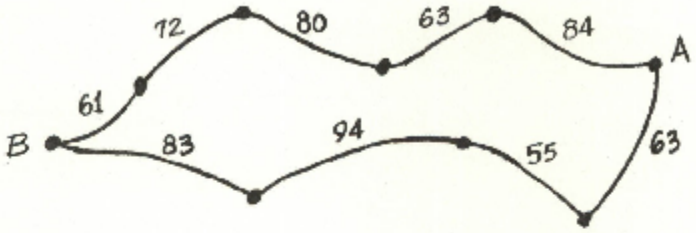
8. _____

(cont)

Mastery Review: (cont)

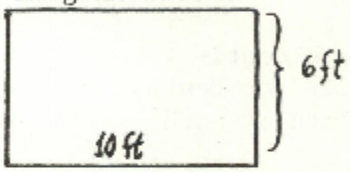
Answers:

9. Based on the diagram below, what was your total mileage if you made the round trip between A and B along the path shown?



9. _____

10. Based on the diagram below:



(a) What is the length of the base of the rectangle?

(b) How long is the height of the rectangle?

(c) What is the perimeter of the rectangle?

10. (a) _____

(b) _____

(c) _____

11. What is the perimeter of a 15 foot by 1 foot rectangle?

11. _____

12. What is the area of a 6 foot by 8 foot rectangle?

12. _____

13. What is the area of a 6 yard by 8 yard rectangle?

13. _____

14. What is the area of a rectangle that is 6 yards by 8 feet? Write the answer in square feet.

14. _____

15. The length of each side of a square is 7 inches. What is:

(a) the perimeter of the square?

(b) the area of the square?

15. (a) _____

(b) _____

16. How many square inches are there in a:

(a) 6-inch square?

(b) 7-inch square?

(c) 8-inch square?

16 (a) _____

(b) _____

(c) _____

17. There are 3 feet per yard. How many square feet are there in 4 square yards?

17. _____

(cont)

Mastery Review (cont)

Answers:

18. The area of a certain square is 1,152 square inches. What is its area in square feet?

18. _____

19. Find the value of:
    (a) 14^2
    (b) 15^2
    (c) 14.5^2

19 (a) _____
(b) _____
(c) _____

20. Rounded off to the nearest whole number, what is the square root of 360? That is, how much is $\sqrt{360}$?

20. _____

21. The area of a square is 360 square centimeters. To the nearest millimeter (or tenth of a centimeter) what is the length of each side of the square?

21. _____

22. The side of a cube is 6 cm long. What is the volume of the cube?

22. _____

23. Starting in the year 1900, the price of a $1 object doubles every ten years. How much did the price of the object increase by during the ten years between:
    (a) 1920 and 1930?
    (b) 1960 and 1970?

23 (a) _____
(b) _____

24. A bank pays an annual interest rate of 8%. How much interest will a deposit of $16,000 earn in one year?

24. _____

25. Under the conditions of Problem 24, how much will the deposit earn during the second year?

25. _____

26. A bank pays an interest rate of 8% compounded annually. If you deposit $16,000 into such an account, how much money will you have at the end of 3 years? Round off the answer to the nearest cent.

26. _____

27. If the diameter of a circle is 20", what is its circumference? Use 3.14 to stand for π.

27. _____

(cont)

Mastery Review: (concluded)

Answers:

28. Using 3.14 for π, what is the diameter of a circle if its circumference is 20 inches? Write your answer rounded off to the nearest hundredth of an inch.

28. _____

29. Using 3.14 for π, what is the area of a circle if its radius is 8 cm?

29. _____

Answers:

1. $6,600 2. (a) $6,300 (b) $63 3. 498

4. 93 5. 360 miles 6. 45 miles per hour

7. 20 8. $24.80 9. 655 miles 10. (a) 10 feet (b) 6 feet (c) 32 feet

11. 32 feet 12. 48 square feet 13. 48 square yards

14. 144 square feet 15. (a) 28 inches (b) 49 square inches

16. (a) 36 (b) 49 (c) 64 17. 36 18. 8 square feet

19. (a) 196 (b) 225 (c) 210.25 20. 19 21. 19.0 cm (190 mm)

22. 216 cubic centimeters (cc) 23. (a) $4 (b) $64

24. $1,280 25. $1,382.40 26. $20,155.39

27. 62.8" 28. 6.37 inches 29. 200.96 square centimeters.

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Step 5:

Do Self-Test 11, Form A on the next page.

Self-Test 11, Form A

ANSWERS:

1. You buy 6 pounds of meat at $2.10 per pound and 4 more pounds at $1.90 per pound. What was the average price per pound?

1. _____

2. You drive from town A to town B, a distance of 60 miles, in 2 hours. You make the return trip in 3 hours.

   (a) What was your average speed in going from A to B?

   (b) What was your average speed for the return trip?

   (c) What was your average speed for the round trip?

2. (a) _____

   (b) _____

   (c) _____

3. Grass seed costs 94¢ per pound. If 1 pound of grass seed covers 150 square feet, how much will it cost for seed, to seed a rectangular lawn that is 25 feet by 60 feet?

3. _____

4. If the area of a square is 3,600 square feet, what is its perimeter?

4. _____

5. The circumference of a circle is 125 inches. To the nearest whole number of square inches, what is the area of this circle? Let $\pi = 3.1416$.

5. _____

6. There are 2.54 centimeters per inch. How many cubic centimeters (c.c.) are there in 3 cubic inches?

6. _____

7. A liter is 1,000 c.c. To the nearest whole number, how many cubic inches are there in a 1.9 liter engine?

7. _____

8. You invest $4,000 at an interest rate of 8% compounded annually. If $1.08^9 \doteq 2$, how long will it take for your investment to grow to a value of $64,000?

8. _____

9. Referring to the previous exercise, if $1.08^8 \doteq 1.85$, how much did your investment earn during the ninth year?

9. _____

10. If an 8% interest rate is compounded *semi-annually*, it means that you get 4% every 6 months (twice a year). If $1.04^{10} \doteq 1.5$, how much money will you have after 5 years if you invest $10,000 at an interest rate of 8% compounded semi-annually?

10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 11, Form A

1. *$2.02*
2. *(a) 30 mph (b) 20 mph (c) 24 mph*
3. *$9.40*
4. *240 feet*
5. *1,243 $quare inches*
6. *49.161192*
7. *116*
8. *36 years*
9. *$600 (approximately)*
10. *$15,000 (approximately)*

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If you did each problem in Self-Test 11, Form A correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 6.

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Step 6:

Study the solutions for Self-Test 11, Form A with special emphasis on any problems you failed to answer correctly.

Solutions for Self-Test 11, Form A

1.

Here we have two constant rates. Namely:

	6 lbs @ \$2.10 = 6 X \$2.10 =	\$12.60
+		
	4 lbs @ \$1.90 = 4 X \$1.90 =	7.60
	10 lbs =	\$20.20

Since 10 pounds cost \$20.20, we find the average cost per pound by dividing \$20.20 by 10 pounds to get:

\$20.20 ÷ 10 pounds = \$2.02 per pound

In other words, 10 pounds @ \$2.02 would add up to \$20.20

Note:

The average of \$2.10 and \$1.90 is the number midway between them; which is \$2.00. But at \$2.00 per pound the total cost of 10 pounds would be \$20.00, not \$20.20.

The thing to keep in mind is that we didn't buy the same number of pounds at each price. In fact 6 of the 10 pounds--or $\frac{3}{5}$ of the pounds--were at \$2.10.

If we bought the same number of pounds at \$2.10 as we did at \$1.90 then the average price per pound would have been \$2.00

The difference between \$1.90 and \$2.10 is 20¢ and $\frac{3}{5}$ of 20¢ = 12¢. If we add the 12¢ to \$1.90 we get \$2.02.

In other words, the more pounds we buy at \$2.10, the closer the average price per pound will be to \$2.10. Similarly, the more pounds we buy at \$1.90 the closer the average price per pound will be to \$1.90.

For example:

8 lb @ \$2.10 =		*\$16.80*
2 lb @ \$1.90 =		*3.80*
10 lb =		*\$20.60*

or \$2.06 per pound

8 lb @ \$1.90 =		*\$15.20*
2 lb @ \$2.10 =		*4.20*
10 lb =		*\$19.40*

or \$1.94 per pound

2.

This is essentially the same idea as what we did in Exercise 1, except it's a different application.

2. (cont)

To find the average speed we take the total distance and divide by the time it took to travel this distance. Therefore:

(a)

We went 60 miles in 2 hours, so our average speed was:

60 miles ÷ 2 hours

or

30 miles per hour

(b)

We're still going 60 miles, but now it takes 3 hours, so our average speed was:

60 miles ÷ 3 hours

or

20 miles per hour

(c)

The round trip is 120 miles (60 miles from A to B plus another 60 miles from B back to A) and the total time is 5 hours (2 hours going and 3 hours coming back). So the average speed for the round trip is:

120 miles ÷ 5 hours

or

24 miles per hour

Note that the average of 20 and 30 is 25, yet the average speed for the round trip is not 25 mph but rather 24 mph. For a further discussion, read the note in the margin.

Note (To be read after reading the solution)

If you travel the same amount of TIME at both speeds, then the average speed would have been the average of the two speeds. For example:

3 hrs @ 20 mph = 60 miles
3 hrs @ 30 mph = 90 miles
6 hrs = 150 miles
or 25 mph

The problem in our exercise is that since the distance from A to B is the same as the distance from B to A, we spend less time at the greater speed. That is, we spent 2 hours @ 30 mph and 3 hours @ 20 mph.

So just as in Exercise 1, we spent 3 of the 5 hours--or 3/5 of the time--at the slower speed (20mph). Hence the average speed for the round trip must be closer to 20 mph that to 30 mph.

In fact, the difference between 20 mph and 30 mph is 10 mph and 3/5 of 10mph is 6 mph. Hence the average speed is 6 mph less than 30 mph (or 4 mph greater than 20 mph); and this shows us why the average speed for the round trip was 24 mph.

Moreover, the answer really doesn't depend on the distance between A and B. That is, 20 mph is 1 mile per 3 minutes and 30 mph is 1 mile per 2 minutes. So each 1 mile round trip means we went 2 miles in 5 minutes or 2 X 12 miles in 5 X 12 minutes or 24 miles per 60 minutes or 24 miles per hour.

3.

It's going to cost 94¢ for every 150 square feet of lawn. Since the lawn is 25 feet by 60 feet, its area is:

25 feet X 60 feet =

(25 X 60) square feet =

1,500 square feet

Since 1,500 is the tenth multiply of 150, we'll need 10 pounds of grass seed. And at 94¢ per pound, 10 pounds will cost

94¢ X 10

or

$9.40

See the difference between area and perimeter? We're placing the seed inside the entire rectangle, not around the boundary. The key comes from the word "square" in the label. If we wanted to put fencing around the boundary, then we'd add 25 feet and 60 feet to get 85 feet. Then we'd double this to get 170 feet (NOT square feet) of fencing.

4.

Since the length and width are equal in a square, we're looking for the number which when multiplied by itself is 3,600. That is, we want $\sqrt{3,600}$ and this is 60.

Since the area is in *square* feet, the length is in feet. Hence the length of each side is 60 feet.

All four sides of a square have equal lengths. So if the length of one side is 60 feet, the perimeter will be 60 feet X 4 or 240 feet.

If you use a calculator, enter 3600 and press the square root key. Without a calculator, notice that

3,600 = 36 X 100

= 6 X 6 X 10 X 10

= (6 X 10) X (6 X 10)

= 60 X 60

If all else fails, keep using trial and error until you get the correct answer.

Again, don't confuse area and preimeter. If the perimeter were 3,600 feet, then we'd divide by 4 to find that the length of each side was 900 feet.

Read carefully! Look at the label. If it says "square feet" it's area. If it says "feet" it's perimeter.

5.

This is a little tricky because it combines two recipes that we've already learned. We know that the circumference (C) and the diameter (D) are related by the formula:

$$C = \pi \times D \qquad (1)$$

while the area (A) and the radius (r) are related by the formula:

$$A = \pi \times r \times r$$

$$= \pi r^2 \qquad (2)$$

We're given the value for C and want to find the value of r. One way to proceed is:

(1) Knowing the value of C we can use formula (1) to find D.

(2) Knowing D, we divide it by 2 to get r.

(3) Knowing r, we can use (2) to find A.

Remember that the length of the diameter is twice the length of the radius.

Let's go!

(1) We replace C by 125 (inches) and π by 3.1416 in formula (1) to get:

$$125 \text{ inches} = 3.1416 \times D$$

or

$$125 \text{ inches} \div 3.1416 = D$$

Once we know the formula, the actual arithmetic is the same as it was in our earlier modules.

Hence

$$D \doteq 39.788643 \text{ inches}$$

You should be using a calculator for this. If you are, you get these 6 digits almost at once. Try to refrain from rounding off until the very last step. Without a calculator the amount of work is prohibitive if we don't round off at once.

(2) Since the radius is half of the diameter we divide our answer in step (1) by 2 to get that the radius is 19.89431 inches.

5. (cont)

(3) Now that we know the radius we use Formula (2). That is:

(i) we square 19.89431 to get (approximately) 395.78402.

We say "approximately" because the calculator display "chops" off the decimal fraction after 8 digits.

(ii) Then we multiply by π in the form 3.1416 to get approximately 1,243.3951

In fact, as soon as we replaced π by 3.1416 we were already making an approximation--as discussed in the Note following the solution.

And since the answer is to be to the nearest whole number, we round it off to obtain:

$A \doteq 1{,}243$ square inches

Note:

If you're using a calculator, it is not much extra work to wait until the last step in the calculations to round off. If you round off too soon, you may induce an error.

For example, suppose you decided that 3.14 was a good enough approximation for π since any answer was going to be rounded off to the nearest whole number. We'd have obtained:

$125 \div 3.14 \doteq 39.81$

$39.81 \div 2 \doteq 19.905$

$19.905^2 \doteq 396.20903$

$3.14 \times 396.20903 \doteq 1{,}244.0963$

$\doteq 1{,}244.$

π = 3.14159.... which makes it closer in value to 3.1416 than to 3.14 so we lose accuracy by replacing 3.1416 by 3.14

The more accurate answer is 1,243. This may not seem like much of an error but it does show that accuracy increases if you round off as late as possible.

6.

A cubic inch is 1" by 1" by 1".

Hence its volume is:

1 inch X 1 inch X 1 inch (1)

We can replace each inch in (1) by 2.54 centimeters (cm) to get:

6. (cont)

2.54 cm X 2.54 cm X 2.54 cm =

2.54^3 cubic centimeters =

16.387064 cm^3

That is there are:

16.387064 cm^3 per 1 in^3 (2)

Since we have 3 cubic inches (3 in^3), we multiply the number of cm^3 in (2) by 3 to get:

3 cubic inches = (16.387064 X 3) cubic centimeters

= 49.161192 cubic centimeters

So notice, again, the value of the calculator. Namely, since there are 2.54 cm per inch, there are 2.54^3 cubic cm per 1 cubic inch, and it is tedious to compute 2.54^3 without a calculator.

We often abbreviate cm X cm X cm by cm^3

In other words:
3 cubic inches =
3 X 1 cubic inch =
3 X 1 inch X 1 inch X 1 inch =
3 X 2.54 cm X 2.54 cm X 2.54 cm

To compute 2.54^3 with a calculator, you could multiply 2.54 by 2.54 and then multiply this product by 2.54. However if you have a "y^x" key, simply enter 2.54, press the "y^x" key, enter 3, then press the "=" key.

7.

From Exercise 6 we know how to convert cm^3 to in^3. So we first convert liters to cubic centimeters by multiplying by 1,000. That is:

$$\frac{1.9 \text{ liters}}{1} \times \frac{1{,}000 \text{ cm}^3}{1 \text{ liter}} = 1.9 \times 1{,}000 \text{ cm}^3$$

$$= 1{,}900 \text{ cm}^3$$

From Exercise 6 we can also write:

$$\frac{1{,}900 \text{ cubic cm}}{1} \times \frac{1 \text{ cubic inch}}{16.387064 \text{ cubic cm}} =$$

(1,900 ÷ 16.387064) cubic inches =

115.94511 cubic inches,

which to the nearest whole number is

116 cubic inches.

Depending on the degree of accuracy desired, you might round 16.387064 to 16.4. In this exercise, the answe wouldn't change. However, since it's easy to retain all the digits with a calculator, the safest thing i to round off as late as possible in any problem.

8.

The fact that $1.08^9 \doteq 2$ means that at an interest rate of 8% compounded annually, $1 will double in about 9 years.

Recall that 1.08 is the decimal form of 108%. So after 1 year you have 1.08 X $1, after 2 years you have 1.08 X (1.08 X $1) or $(1.08)^2$ and so on. Hence after n years $1 is worth $(1.08)^n$. In particular after 9 years, the $1 is worth $(1.08)^9$ or approximately $2.

What will $4,000 do? Well, we may view $4,000 as 4,000 separate $1 investments. So if each $1 doubles in 9 years, so will the entire $4,000.

At the end of 9 years, the interest is applied to the new (doubled) amount. That is, every 9 years the amount we started with during that 9-year period doubles.

In other words, in about 9 years the $4,000 becomes $8,000. At the end of the next 9 years (that is, 18 years after the $4,000 is deposited) the $8,000 becomes $16,000 and so on. That is:

At the beginning:	$ 4,000
After 9 years:	$ 8,000
After 18 years:	$16,000
After 27 years:	$32,000
After 36 years:	$64,000

Since the money doubles each 9 years, we look at the years that are multiples of 9; that is, 9, 18, 27, and 36. During the next 9 years the $64,000 doubles, so that after 45 years you have $128,000. This is what retirement funds are all about.

Note 1:

Do you see this is in effect a non-constant rate? During the first 9 years, the value of your original investment ($4,000) doubled. But during the fourth 9-year period, your investment increased from $32,000 to $64,000, which is a $32,000 increase. This is 8 times your original investment.

From another point of view you gained $60,000 (that is, $64,000 - $4,000). This is an average of $60,000 ÷ 36 or $1,666.67. Based on an investment of $4,000 you've earned an average of $1,666.67 per $4,000 or $41\frac{2}{3}$%--a far cry from 8%.

8. (cont)

Note 2:

If we hadn't been told that $1.08^9 \doteq 10$, we could have worked it out for ourselves on a calculator. Based on a $1 investment we have:

After 1 year: $\$(1.08)^1$ or $1.08

After 2 years: $\$(1.08)^2$ or $1.1664

After 3 years: $\$(1.08)^3$ or $1.259112

After 4 years: $\$(1.08)^4$ or $1.360489

After 5 years: $\$(1.08)^5$ or $1.4693281

After 6 years: $\$(1.08)^6$ or $1.5868743

After 7 years: $\$(1.08)^7$ or $1.7138243

After 8 years: $\$(1.08)^8$ or $1.8509320

After 9 years: $\$(1.08)^9$ or $1.9990046

Each year we multiply that amount by 1.08 to find what we have the next year.

We don't round off yet to the nearesnt cent, because if we started, for example, with $100,000; we'd have to multiply the value of $1 by 100,000. This involves moving the decimal point 5 places to the right. If we multiply, say, $1.259112 by 100,000 we get $125,911.20. But if we have rounded off to $1.26, the answer would have been $126,000; which is off by $88.80

This is why we said that $1.08^9 \doteq 2$. So if we wanted a more exact answer, after 9 years we would have 1.9990046 X $4,000 or approximately $7,996.02, rather than $8,000. Hence it would take a bit longer than 36 years to have $64,000.

Note 3:

If your calculator has a "y^x" key there is a quicker way to compute 1.08^9. Namely:

Enter 1.08

Press the "y^x" key

Enter 9

Press the "=" key.

The display then shows the value of 1.08^9.

In fact we can use this key to find, quite quickly, how much money you'd have at the end of 36 years. Namely:

Enter 1.08

Press the "y^x" key

Enter 36

Press the "=" key (This tells you what $1 has become afer 36 years)

$1.08^{36} \doteq 15.968172$. So after 36 years, $4,000 has grown to $4,000 X 15.968172 or $63,872.69 (to the nearest cent)

Press the "X" key

Enter 4,000

Press the "=" key (This is what $4,000 is worth)

9.

The fact that $1.08^9 \doteq 2$ tells us that we have approximately \$4,000 X 2 or \$8,000 at the end of 9 years.

The fact that $1.08^8 \doteq 1.85$ tells us that we have approximately \$4,000 X 1.85 or \$7,400 at the end of 8 years.

So during the 9th year we added on to \$7,400 the amount necessary to get \$8,000. That is, *during the 9th year*, our \$4,000 investment earned:

\$8,000 - \$7,400

or

\$600

To get a more precise answer, we could use our table in the solution to Exercise 8 to get:

After 9 years: \$4,000 X 1.9990046 ≐ \$7,996.02

- After 8 years: \$4,000 X 1.8509320 ≐ \$7,403.73

During the 9th year: ≐ \$ 592.29

The key to this exercise is making sure you understand the difference between "during nine years" and "during THE nineTH year"

This idea always holds. For example, to see what you earned during the 36th year, you find out how much you had at the end of 36 years; and then subtract the amount you had at the end of 35 years.

In terms of a percent, you earned \$600 per \$4,000 or 600/4000 X 100% or 15%. See how your annual growth is incresing? The rate for the 9th year is almost double that of the first year.

10.

Giving half the rate twice as often is a way of making an investment more attractive. See the idea? If you start with \$1 after 6 months you have \$1.04. But during the next 6 months you're earning 4% of the entire \$1.04. So by comparison:

8% compounded semi-annually:	8% semi-annually
After 1 year \$1 becomes \$1.08	After 6 months, \$1 becomes \$1.04. After another 6 months you have $\$(1.04)^2$ or \$1.0816

If the rate were quarterly, you'd get 1/4 of the rate 4 times a year. So at the end of 1 year you'd earn 2% four times. That is, you'd have:

$\$(1.02)^4 \doteq \$1.0824322$

The more often the interest is paid, the faster your deposit grows.

10. (cont)

So, since you're earning 4% (half of 8%) but now twice each year; after five years you have:

$$\$10{,}000 \times 1.04^{10} \qquad (1)$$

In 5 years you get interest 10 times; that is 2 times in each of the 5 years.

Since we're told that

$$1.04^{10} \doteq 1.5$$

we may replace 1.04^{10} by 1.5 in (1) to get:

$$\$10{,}000 \times 1.5$$

or \$15,000

We can get the exact value of 1.04^{10} by entering 1.04, pressing the "y^x" key, entering 10, and pressing the "=" key. We'd get 1.480244. Hence a more accurate answer would be \$14,802.44. But the given estimate is sufficient for making the point in this exercise.

The answer is approximate because our value for 1.04^{10} was approximate.

Note that if the interest had been compounded annually, the value after 5 years would be:

$$\$10{,}000 \times 1.08^5 \doteq$$

$$\$10{,}000 \times 1.4693281 \doteq$$

We had this result in our solution of Exercise 8

$$\$14{,}693.28$$

Also note that if we took the 8% rate and multiplied it by 5, we'd get 40%. A 40% increase on \$10,000 would have given us \$14,000 (that is, 140% of \$10,000). So here again we see the importance of exponential growth. Compounding interest makes investments grow much faster than you might expect

Step 7:

Do Self-Test 11, Form B on the next page.

Self-Test 11, Form B

ANSWERS:

1. You buy 7 pounds of meat at $2.40 per pound and 3 more pounds at $2.10 per pound. What was the average price per pound for the 10 pounds of meat?

1. _____

2. You drive from town A to town B, a distance of 105 miles, in 3 hours. You make the return trip in 7 hours.

   (a) What was your average speed in going from A to B?

   (b) What was your average speed for the return trip?

   (c) What was your average speed for the round trip?

2. (a) _____

   (b) _____

   (c) _____

3. Grass seed costs 89¢ per pound. If 1 pound of grass seed covers 175 square feet, how much will it cost for seed to seed a rectangular lawn that is 70 feet by 15 feet?

3. _____

4. If the area of a square is 4,900 square centimeters, what is its perimeter?

4. _____

5. The circumference of a circle is 200 inches. To the nearest whole number of square inches, what is the area of this circle? Let $\pi = 3.1416$

5. _____

6. If there are 2.54 centimeters per inch, how many cubic centimeters (c.c.'s) are there in 4 cubic inches? Write your answer to the nearest whole number of cubic centimeters.

6. _____

7. A liter is 1,000 c.c.'s. To the nearest whole number, how many cubic inches are there in a 2.4 liter engine?

7. _____

8. You invest $6,000 at an interest rate of 6% compounded annually. If $1.08^{12} \doteq 2$, how long will it take for your investment to grow to a value of $48,000?

8. _____

9. Referring to the previous exercise, if $1.06^{11} \doteq 1.9$, how much did your investment earn during the 12th year?

9. _____

10. If a 6% interest rate is compounded semiannually, it means that you get 3% every 6 months. If $1.03^{12} \doteq 1.43$, how much money will you have after 6 years if you invest $6,000 at an annual interest rate of 6% compounded semiannually?

10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 11, Form B

1. *$2.31*
2. *(a) 35 mph (b) 15 mph (c) 21 mph*
3. *$5.34*
4. *280 centimeters*
5. *3,183 square inches*
6. *66 cubic centimeters*
7. *146 cubic inches*
8. *approximately 36 years*
9. *approximately $600 (A more accurate amount is $683.39)*
10. *approximately $8,580*

****

If you did each problem in Self-Test 11, Form A correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8:

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Step 8:

View the solutions for Self-Test 11, Form B on Videotape Lecture 11S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. Feel free to rewind the tape at any time to restudy any problems that still give you difficulty.

Step 9:

Do Self-Test 11, Form C on the next page.

Self-Test 11, Form C

ANSWERS:

1. You buy 8 pounds of meat at $2.50 per pound and 2 more pounds at $3.10 per pound. What was the average price per pound for the 10 pounds of meat?

1. _____

2. You drive from town A to town B, a distance of 90 miles in 2 hours. You make the return trip in 3 hours.

   (a) What was your average speed in going from A to B?

   (b) What was your average speed for the return trip?

   (c) What was your average speed for the round trip?

2. (a) _____

   (b) _____

   (c) _____

3. Grass seed costs 97¢ per pound. 1 pound of grass covers 240 square feet. How much will it cost for seed to seed a rectangular lawn that is 300 feet by 32 feet?

3. _____

4. If the area of a square is 6,400 square yards, what is its perimeter?

4. _____

5. The circumference of a circle is 180 feet. To the nearest whole number of square feet, what is the area of this circle. Let $\pi = 3.1416$

5. _____

6. If there are 2.54 centimeters per inch, to the nearest whole number of cubic centimeters, how many cubic centimeters are there in 5 cubic inches?

6. _____

7. A liter is 1,000 cubic centimeters. To the nearest whole number, how many cubic inches are there in a 2.9 liter engine?

7. _____

8. You invest $8,000 at an interest rate of 7.2%. compounded annually. If $1.072^{10} \doteq 2$, how long will it take for your investment to grow to a value of $64,000?

8. _____

9. Referring to the previous exercise, if $1.072^{9} \doteq 1.87$ how much did your investment earn during the 10th year?

9. _____

10. If a 10% interest rate is compounded semi-annually, it means that you get 5% twice each year. If $1.05^{6} \doteq 1.34$, how much money will you have after 3 years if you invest $10,000 at an annual interest rate of 10% compounded semiannually?

10. _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 11, Form C

1. *$2.62*
2. *(a) 45 mph (b) 30 mph (c) 36 mph*
3. *$38.80*
4. *320 yards*
5. *2,578 square feet*
6. *82 cubic centimeters*
7. *177 cubic inches*
8. *approximately 30 years*
9. *approximately $1,040 ($1,076.90 is a closer estimate)*
10. *approximately $13,400*

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THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #11.

HOPEFULLY, YOU WILL FEEL READY TO BEGIN MODULE #12.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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